Additional Operators

These operators do not add any power (expressiveness) to the relational algebra but simplify common (often complex and lengthy) queries.

Set-Intersection	$\cap$	
Natural Join	$\bowtie$	
Condition Join	$\bowtie_C$	(also called Theta-Join)
Division	$\div$	
Assignment	$\longleftarrow$	

Set-Intersection

- Notation: $r \cap s$
  Defined as $r \cap s := \{t \mid t \in r \textbf{ and } t \in s\}$
- For $r \cap s$ to be applicable,
  1. r and s must have the same arity
  2. Attribute domains must be compatible
- Derivation: $r \cap s = r - (r - s)$
- Example: given the relations r and s

r

A	B
α	1
α	2
β	1

s

A	B
α	2
β	3

$r \cap s$

A	B
α	2

Natural Join

- Notation: $r \bowtie s$
- Let r, s be relations on schemas R and S, respectively. The result is a relation on schema $R \cup S$. The result tuples are obtained by considering each pair of tuples $t_r \in r$ and $t_s \in s$.
- If t_r and t_s have the same value for each of the attributes in $R \cap S$ ("same name attributes"), a tuple t is added to the result such that
  - t has the same value as t_r on r
  - t has the same value as t_s on s
- Example: Given the relations $R(A, B, C, D)$ and $S(B, D, E)$
  - Join can be applied because $R \cap S \neq \emptyset$
  - the result schema is (A, B, C, D, E)
  - and the result of $r \bowtie s$ is defined as

$$\pi_{r.A,r.B,r.C,r.D,s.E}(\sigma_{r.B=s.B \wedge r.D=s.D}(r \times s))$$

- Example: given the relations r and s

r

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

s

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	τ

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

Condition Join

- Notation: $r \bowtie_C s$

  C is a condition on attributes in $R \cup S$, result schema is the same as that of Cartesian Product. If $R \cap S \neq \emptyset$ and condition C refers to these attributes, some of these attributes must be renamed.
  Sometimes also called *Theta Join* ($r \bowtie_\theta s$).

- Derivation: $r \bowtie_C s = \sigma_C(r \times s)$

- Note that C is a condition on attributes from both r and s

- Example: given two relations r, s

r

A	B	C
1	2	3
4	5	6
7	8	9

s

D	E
3	1
6	2

$r \bowtie_{\text{B<D}} s$

A	B	C	D	E
1	2	3	3	1
1	2	3	6	2
4	5	6	6	2

If C involves only the comparison operator "$=$", the condition join is also called *Equi-Join.*

- Example 2:

r

A	B	C
4	5	6
7	8	9

s

C	D
6	8
10	12

$r \bowtie_{\mathsf{C=SC}} (\rho_{\mathsf{S(SC,D)}}(s))$

A	B	C	SC	D
4	5	6	6	8

Division

- Notation: $r \div s$
- Precondition: attributes in S must be a subset of attributes in R, i.e., $S \subseteq R$. Let r, s be relations on schemas R and S, respectively, where
  - $R(A_1, \ldots, A_m, B_1, \ldots, B_n)$
  - $S(B_1, \ldots, B_n)$

  The result of $r \div s$ is a relation on schema
  $R - S = (A_1, \ldots, A_m)$
- Suited for queries that include the phrase "for all".

  The result of the division operator consists of the set of tuples from r defined over the attributes $R - S$ that match the combination of **every** tuple in s.

  $r \div s := \{t \mid t \in \pi_{R-S}(r) \wedge \forall u \in s \colon tu \in r\}$

- Example: given the relations r, s:

r

A	B	C	D	E
α	a	α	a	1
α	a	γ	a	1
α	a	γ	b	1
β	a	γ	a	1
β	a	γ	b	3
γ	a	γ	a	1
γ	a	γ	b	1
γ	a	β	b	1

s

D	E
a	1
b	1

$r \div s$

A	B	C
α	a	γ
γ	a	γ